# $H$-cluster stars

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The state of cold matter at supra-nuclear density depends on the non-perturbative nature of quantum chromo-dynamics (QCD) and is essential for modeling pulsars. In compact stars at only a few nuclear densities and extremely low temperature, quarks could be interacting strongly with each other there. That might render quarks grouped in clusters, although the hypothetical quark-clusters in cold dense matter has not been confirmed due to the lack of both theoretical and experimental evidence. Motivated by recent lattice QCD simulations of the $H$-dibaryons (with structure $uuddss$), we are considering here a possible kind of quark-clusters, $H$-clusters, that could emerge inside compact stars during their initial cooling, as the dominant components inside. We study the stars composed of $H$-clusters, i.e., $H$-cluster stars, and derive the dependence of their maximum mass on the potential of $H$-$H$ interaction and the in-medium stiffening effect, showing that the maximum mass could be well above 2 $M_\odot$ under reasonable parameters. Besides a general understanding of different manifestations of compact stars, we expect further observational and experimental tests for $H$-cluster stars in the future.

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## I. INTRODUCTION

The study of pulsar-like compact stars opens a unique window that relates fundamental particle physics and astrophysics. At average density higher than $\sim 2$ times nuclear matter density $\rho_0$, the quark degrees of freedom inside would not be negligible, and such compact stars are then called quark stars [1, 2]. Bodmer-Witten conjecture says that strange quark matter (composed of $u$, $d$ and $s$ quarks) could be more stable than nuclear matter [3, 4]. Although the effect of non-perturbative QCD makes it difficult to derive the real state of cold quark matter, the existence of quark stars cannot be ruled out neither theoretically nor observationally (see a review in [5]).

Although quark matter at high density and low temperature is difficult to be created in laboratory as well as difficult to be studied by QCD calculations, some efforts have been made to understand the state of cold quark matter and quark stars. MIT bag model treats the quarks as relativistic and weakly interacting particles, which is the most widely used model for quark stars [2]. The color super-conductivity (CSC) state is currently focused on under perturbative QCD as well as QCD-based effective models [6]. In most of these models, quark stars are characterized by soft equations of state, because the asymptotic freedom of QCD tells us that as energy scale goes higher, the interaction between quarks becomes weaker.

In cold quark matter at realistic baryon densities of compact stars ($\rho \sim 2 - 10\rho_0$), however, the energy scale is far from the region where the asymptotic freedom approximation could apply. In this case, the interaction energy between quarks could be comparable to the Fermi energy, so the the ground state of realistic quark matter might not be that of Fermi gas (see a discussion given in [7]). Some evidence in heavy ion collision experiments shows also that the interaction between quarks is still very strong even in the case of hot quark-gluon plasma [8]. It is then reasonable to infer that quarks could be coupled strongly also in the interior of those speculated quark stars, which could make quarks to condensate in position space to form quark clusters. The observational tests from polarization, pulsar timing and asteroseismology have been discussed [9], and it is found that the idea of clustering quark matter could provide us a way to understand different manifestations of pulsars. The realistic quark stars could then be actually "quark-cluster stars" [10]. An interesting suggestion is that quark matter could be in a solid state [11–13], and for quark-cluster stars, solidification could be a natural result if the kinetic energy of quark clusters is much lower than the interaction energy between the clusters.

Quark clusters may be analogized to hadrons, and in fact some authors have discussed the stability of hadron bound states. A dihyperon with quantum numbers of $\Lambda\Lambda$ ($H$ dibaryon) was predicted to be a stable state or resonance [14], and an 18-quark cluster (quark-alpha, $Q_\alpha$) being completely symmetric in spin, color and flavor spaces was also proposed [15]. $H$ dibaryon in lattice QCD simulations, although no direct evidence from experiments, provides us a specific kind of quark clusters that could be very likely to exist inside quark stars. In fact, $H$ dibaryons have been studied for years as a possible kind of multi-quark compound states. The non-relativistic quark-cluster model was introduced to study the binding energy of $H$-clusters [16]. The interaction between $H$-clusters was investigated by employing one-gluon-exchange potential and an effective meson exchange potential, and a short-range repulsion was found [17]. Recently, $H$ dibaryon, with binding energy of about 10 to 40 MeV, has been found in lattice QCD simulations by two independent groups [18, 19], and STAR preliminary results show also possible stable $H$ dibaryons

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from Λ-Λ correlations (Huanzhong Huang, private communications).

Strange quark matter, with light flavor symmetry, has nearly equal numbers of $u$, $d$ and $s$ quarks. If quark-clusters are the dominant components of strange quark matter, then it is natural to conjecture that each quark-cluster could composed of equal numbers of $u$, $d$ and $s$ quarks. During the initial cooling of a quark star, the interaction between quarks will become stronger and stronger, then $H$-clusters (six-quark clusters with the same structure as $H$ dibaryons $uuddss$) would emerge from the combination of three-quark clusters (with structure $uds$ as $\Lambda$ particles), due to the attraction between them. If the light flavor symmetry is ensured, then the dominant components inside the stars is very likely to be $H$-clusters. In our previous work about the quark-clusters stars, the number of quarks inside each quark-cluster $N_q$ is taken to be a free parameter [20], and as the further study in this paper, we realistically specify the quark-clusters to be $H$-clusters.

There could be other kind of particles with strangeness, such as kaons and hyperons. Kaon condensate would probably reduce the maximum mass of the stars and hyperons heavier than $\Lambda^0$ would not have large enough number densities, both of which would not have significant effect on the stars' global structure. We neglect them in this paper as the first step towards the structure of this specific kind of quark-cluster stars, and the effect of all possible particles should be taken into account in our further studies.

To study the $H$-cluster stars, we assume that the interaction between $H$-clusters is mediated simply by $\sigma$ and $\omega$ mesons and introduce the Yukawa potential to describe the $H$-$H$ interaction [21], and then derive the dependence of the maximum mass of $H$-cluster stars on the depth of potential well, taking into account the in-medium stiffening effect. Under a wide range of parameter-space, the maximum mass of $H$-cluster stars can be well above $2M_\odot$, so they cannot be ruled out even though some pulsars with mass as high as $2M_\odot$ are found [22]. Moreover, the observations (e.g. pulsar-mass) could help us constrain the $H$-$H$ interaction in dense matter.

The paper is arranged as follows. The existence and localization of $H$-clusters inside compact stars are discussed in §II. The equation of state and the global structure of $H$-cluster stars are given in §III, including the dependence of their maximum mass on the $H$-$H$ interaction and the in-medium stiffening effect. Some issues about the $H$-cluster stars are discussed in §IV, and we make conclusions in §V.

## II. $H$-CLUSTERS INSIDE COMPACT STARS

The state of matter of compact stars is essentially a problem of non-perturbative QCD, with energy scale below 0.8 GeV (corresponding to mass density below $10\rho_0$). If the interaction between quarks could be strong enough to group them into clusters, the quark-clustering phase should be very different from the CSC phase under perturbative QCD, and it could also different from the normal hadron phase if the quark matter has light flavor symmetry. Whether $H$-clusters could be the dominant component inside compact stars is an interesting but difficult problem, and here we just make some rough estimation about their existence and quantum effect. We find that at high densities $H$-cluster matter could be more stable than nucleon matter, and could exist against the energy fluctuation. Moreover, they could be localized other than that of Bose-Einstein condensation, and such a localization of quark-clusters could lead to a crystalline structure in solid state.

### A. The stability of $H$-cluster matter

Whether the Bodmer-Witten conjecture is true or not is essentially a non-perturbative QCD problem and is hard for us to solve from first-principle. Here we propose that $H$-cluster matter could be stable with respect to transforming into nucleon matter at the same density, by assuming the Brown-Rho scaling. In dense nuclear matter, the masses of nucleons and mesons satisfy the scaling law $m_N^*/m_N = m_M^*/m_M$, where the masses with and without asterisks stand for in-medium values and free-space values respectively. This is called Brown-Rho scaling [23]. We suppose that the Brown-Rho scaling holds for $H$-dibaryons, and $H$-clusters and nucleons have the same mass-scaling: $m_H^*/m_H = m_N^*/m_N = \eta$ [24]. For the system composed of nucleons in weak equilibrium at densities higher than $\rho_0$, the dominant component is neutrons. Ignoring the potential well of $H$-$H$ interaction (the explicit form of interaction and the depth of potential well will be discussed in the §III), to ensure the stability of H-matter we should have $m_H^*/2 < \sqrt{p_n^2 + m_n^{*2}} + E_{sym}$, where $p_n = (3\pi^2\rho/m_n^*)^{1/3}$ is the Fermi momentum of neutrons, $E_{sym} \simeq 25$ MeV is the symmetry energy per baryon, and we set the mass of $H$-cluster, $m_H = 2m_\Lambda - 20$ MeV $= 2210$ MeV. This inequality holds if $\eta \lesssim 0.84$ at density $\rho = 2\rho_0$. Here we choose $E_{sym}$ to be the value at density $\rho_0$, and at density $2\rho_0$, $E_{sym}$ could be as twice as the value we choose [25]. The contribution of $E_{sym}$ and the potential well of $H$-$H$ interaction could certainly enhance the stability of $H$-cluster matter.

$H$-clusters could also be safe under the high momentum fluctuation $\Delta p$ at high densities inside compact stars, because the energy fluctuation $\Delta E$ is not so high due to their high mass. We can make the estimation of $\Delta E \sim \Delta p^2/2m_H \simeq 7\ \mathrm{MeV}(\rho/10\rho_0)^{2/3}(m_H/2210\ \mathrm{MeV})^{-5/3}$, where $m_\Lambda$ is the mass of $\Lambda^0$. The energy of $\Delta E$ could be not much lower than the binding energy of $H$-clusters and potential drop of interaction between $H$-clusters (see §III.A), but it could be reasonable to ensure the existence of $H$-clusters with large enough mass fraction of the star. The depen-

dence of binding energy of quark-clusters on density is still unknown, but if the mass of $H$-dibaryons decreases with increasing densities like baryons and mesons, this could be equivalent to the increasing of binding energy of $H$-dibaryons and make $H$-clusters to be more stable.

Inside the stars, the repulsion between $H$-clusters becomes stronger and stronger to resist gravity, and then their potential energy would become even positive. It is worth mentioning that, this potential energy does not influence the stability of $H$-clusters inside stars. The pressure equilibrium between repulsion caused by interaction of particles and attraction caused by local gravity ensures that, the energy per particle increased by inter-cluster potential compensate that decreased by local gravitational potential, so the inter-cluster potential will not cause instability. In other words, if $H$-clusters are stable against decaying to lighter particles at the surface of the star, inside the whole star $H$-clusters could be all stable.

### B. Crystallization of $H$-cluster matter

Under the interaction, $H$-clusters could be localized and behave like classical particles. In the core of a neutron star, $H$-clusters could also appear, and the existence of $H$-clusters inside neutron stars has been studied in relativistic mean-field theory [21]. It was found that when the potential between $H$-clusters is negative enough, then a substantial number density of $H$-clusters will reduces the maximum mass of neutron stars if Bose-Einstein condensation happens [26].

However, due to the strong interaction, $H$-clusters would be localized like classical particles in crystal lattice, and the quantum effect would be negligible. One $H$-cluster is under the composition of interaction from its neighbor $H$-clusters, which forms a potential well. The energy fluctuation makes this $H$-cluster oscillate about its equilibrium position with the deviation $\Delta x$, $\Delta E \simeq \hbar^2/(m_H \Delta x^2) \simeq k\Delta x^2$, where $k \simeq \partial^2 V(r)/\partial r^2$, and $r$ is the distance of two neighbor $H$-clusters. We use the $H$-$H$ interaction in Eq(1), and estimate $\Delta x$ at density $\rho = 10\rho_0$, $\Delta x \simeq (\hbar^2/m_H k)^{1/4} \simeq 0.27$ fm$(2210$ MeV$/m_H)^{1/4}$. On the other hand, the distance between two nearby $H$-clusters is $d = n^{-1/3} \simeq 1.1$ fm $(\rho/(10\rho_0))^{-1/3}$, with $n$ the number density of $H$-clusters. Consequently, the interaction would localize $H$-clusters in the potential well at the stellar center, since $\Delta x < d$, and the Bose-Einstein condensate would not take place. On the stellar surface, $\rho \simeq 2\rho_0$, we have $\Delta x = 0.9$ fm and $d \sim 1.9$ fm. The conclusion will not change even if the $m_H$ reduce to half of 2210 MeV.

$H$-clusters are localized because each of them feels an ultra-strong repulsion every direction around it, and such localization could lead to a crystalline structure. In fact, the relation between hard-core potential and crystallization was discussed previously (e.g. [27]). The almost infinitely strong repulsion is certainly an ideal case, but in real world the short range of H-H interaction could be still strong enough to localize them.

## III. THE GLOBAL STRUCTURE OF $H$-CLUSTER STARS

We propose a possible kind of quark-cluster stars totally composed of $H$-clusters, i.e., $H$-cluster stars. $H$-cluster stars could have different properties from neutron stars and conventional quark stars, such as the radiation properties, cooling behavior and global structure. In this paper, we only focus on the global structure of $H$-cluster stars, deriving the mass-radius relation based on the equation of state.

### A. $H$-$H$ interaction and equation of state

The interaction between $H$-clusters has been studied under the Yukawa potential with $\sigma$ and $\omega$ coupling [21], and we adopt this form of interaction here

$$V(r) = \frac{g_{\omega H}^2}{4\pi}\frac{e^{-m_\omega r}}{r} - \frac{g_{\sigma H}^2}{4\pi}\frac{e^{-m_\sigma r}}{r}, \tag{1}$$

where $g_{\omega H}$ and $g_{\sigma H}$ are the coupling constants of $H$-clusters and meson fields. The numerical result of the potential between two $H$-dibaryons shows a minimum at $r_0 \approx 0.7$ fm with the depth $V_0 \approx -400$ MeV [17], which means that, to get the minimal point, two $H$-dibaryons should be very close to each other. To prevent the existence of $H$-dibaryons in normal nuclear matter, $V(\rho_0) \gtrsim -350$ MeV [26].

Nevertheless, the medium effect in dense matter could change those properties. In dense nuclear matter, the effective meson masses $m_M^*$ satisfy the Brown-Rho scaling law [23] $m_M^* \simeq m_M(1 - \alpha_{BR} n/n_0)$, where $\alpha_{BR}$ is the coefficient of the scaling, $m_M$ is the meson mass in free space. The value of $\alpha_{BR}$ is found to be about 0.2 at the nuclear matter density. In the problem we are now considering, however, a quark-cluster star is at supra-nuclear density, and we then use a modified scaling law of

$$m_M^* = m_M \exp(-\alpha_{BR} n/n_0), \tag{2}$$

which also shows the in-medium effect that stiffens the inter-particle potential by reducing the meson effective masses, and approximately the same as the usual scaling law at the nuclear matter density. In this case, $m_\sigma$ and $m_\omega$ in Eq.(1) should be replaced by $m_\sigma^*$ and $m_\omega^*$, which makes $r_0$ and $V_0$ become larger.

Given the potential between two $H$-clusters, we can get the energy density by taking into account all of the contributions from $H$-clusters in the system. Note that in this problem, the interaction between $H$-clusters is mediated by $\sigma$ and $\omega$ mesons, so the interaction at long distance is negligible, even if the mass-scaling effect is considered. Therefore, we only consider the contributions of the nearby particles. In the case of a strong repulsive

core, each $H$-cluster could be trapped inside the potential well as demonstrated before. Assuming the localized $H$-clusters form lattice structure, the interaction energy density could be written as $\epsilon_I \propto nV$. Combining with Eq.(1), we can get the interaction energy density $\epsilon_I$ as a function of $n$,

$$\epsilon_I \propto n^{4/3} \left( \frac{g_{\omega H}^2}{4\pi} e^{-m_\omega^* n^{-1/3}} - \frac{g_{\sigma H}^2}{4\pi} e^{-m_\sigma^* n^{-1/3}} \right), \quad (3)$$

and the pressure is thus

$$P = n^2 \frac{d}{dn} \left( \frac{\epsilon_I}{n} \right). \quad (4)$$

If we know the surface $H$ number density $n_s$ and the depth of the potential well $V_0$, we can determine $g_{\omega H}$ and $g_{\sigma H}$, because at the surface of stars the potential reaches its minimal value: $P(n = n_s) = 0$ and $V(n = n_s) = V_0$. Considering the uncertainty of the interaction, we take $V_0$ as a parameter, and fix the surface density $\rho_s$ to be $2\rho_0$. In addition, we find that different values of $m_H$ do not influence the equation of state significantly.

It is worth noting that, although composed of $H$-clusters, $H$-cluster stars are self-bound. They are bound by the interaction between quark-clusters (the $H$-clusters here). This is different from but similar to the traditional MIT bag scenario. The interaction between $H$-clusters could be strong enough to bind the star, and on the surface, the quark-clusters are just in the potential well of the interaction, leading to non-vanishing density but vanishing pressure.

Compact stars composed of pure $H$-clusters are electric neutral, but in reality there could be some flavor symmetry breaking that leads to the non-equality among $u$, $d$ and $s$, usually with less $s$ than $u$ and $d$. The positively charged quark matter is necessary because it allows the existence of electrons that is crucial for us to understand the radiative properties of pulsars. The pressure of degenerate electrons is negligible compared to the pressure of $H$-clusters, so the contribution of electrons to the equation of state is negligible.

### B. Mass-radius relation of $H$-cluster stars

In general relativity, the hydrostatic equilibrium condition in spherically symmetry is [28]

$$\frac{1 - 2Gm(r)/c^2 r}{P + \rho c^2} r^2 \frac{dP}{dr} + \frac{Gm(r)}{c^2} + \frac{4\pi G}{c^4} r^3 P = 0, \quad (5)$$

where

$$m(r) = \int_0^r \rho \cdot 4\pi r'^2 dr', \quad (6)$$

with $\rho = \epsilon_I / c^2 + n\, m_H$. Inserting the equation of state $P(\rho)$ we can get the total mass $M$ and radius $R$ of an $H$-cluster star by numerical integration. Figure 1 shows the mass-radius and mass-central density (rest-mass energy density) curves, in the case $\rho_s = 2\rho_0$ and $\alpha_{BR}$=0.2, including $V_0 = -10$ MeV (solid line) and $V_0 = -100$ MeV (dashed line). At first, $M$ grows larger as central density increases, and eventually $M$ reaches the maximum value, after which the increase of central density leads to gravitational instability. In the figure, both curves have maximum masses higher than $2M_\odot$ In the calculations, we find that the influence of mass of $H$-clusters, $m_H$, on the results is very tiny and could be negligible.

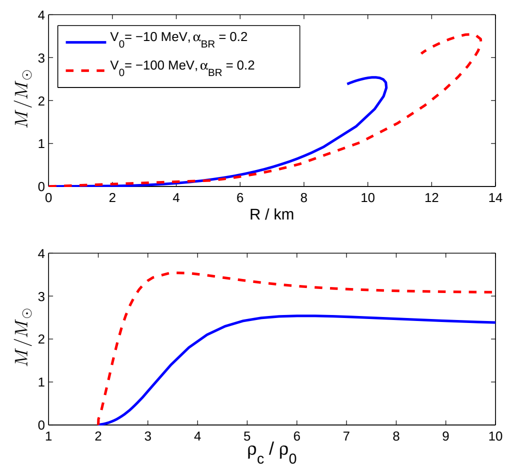


FIG. 1. (Color online) The mass-radius curves and mass-central density (rest-mass energy density) curves, in the case $\rho_s = 2\rho_0$ and $\alpha_{BR}$=0.2, including $V_0 = -10$ MeV (solid line) and $V_0 = -100$ MeV (dashed line).

The observed masses of pulsars put constraints on the state of quark matter. Quark stars have been characterized by soft equations of state, because in conventional quark star models (e.g. MIT bag model) quarks are treated as relativistic and weakly interacting particles. Recently, radio observations of a binary millisecond pulsar PSR J1614-2230 imply that the pulsar mass is $1.97 \pm 0.04\ M_\odot$ [29]. Although we could still obtain high maximum masses under MIT bag model by choosing suitable parameters [30], a more realistic equation of state in the density-dependent quark mass model (e.g., [31]) is very difficult to reach a high enough maximum stellar mass, which was considered as possible negative evidence for quark stars [32]. Nevertheless, some other models of stars with quark matter could be consistent with the observation of the high mass pulsar, such as color-superconducting quark matter model [33] and hybrid star models[6, 34]. Moreover, quark-cluster stars could also have maximum mass $M_{\max} > 2M_\odot$ because of stiff equation of state [20, 22, 35].

In this paper, we study compact stars composed of $H$-clusters and apply the potential model which is widely used in nuclear physics to describe the $H$-$H$ interaction. The depth $V_0$ and position $r_0$ of potential well should be

meaningful for study the properties of cold quark matter with $H$-clusters. The coefficient of Brown-Rho scaling $\alpha_{BR}$ is also unknown in quark matter, whose value could be different from that used in nuclear matter.

The real state of matter at densities of compact stars is essentially a non-perturbative QCD problem and thus hard to solve. We make a phenomenological model that quarks could be grouped into quark-clusters at this energy scale to propose the quark-cluster stars, and specify quark-clusters in this paper to be $H$-clusters. The color super-conductivity model is the most often used one for modeling quark matter, but it is still uncertain that whether the interaction between quarks is weak enough to make the non-perturbative treatment to be reasonable. The point we put in this paper is that, under the assumption of light flavor symmetry, $H$-clusters could be the possible kind of quark clusters, and as a specific quark-cluster stars, $H$-cluster stars could not be ruled out by the observed high mass pulsars.

### C. Maximum mass of $H$-cluster stars

We constrain the parameters $V_0$ and $\alpha_{BR}$ in the context of $H$-cluster stars by the maximum mass of pulsars $M_{\max}$, shown in Figure 2. The interaction between $H$-dibaryons was studies previously and the related parameters were derived by fitting data in experiments of nucleon-nucleon interaction and hypernucleus events (e.g. see [17] and references therein); however, whether the two-particle interaction data are adequate in determining the properties of quark matter is uncertain. Our model for $H$-$H$ interaction could provide us another way to study the properties of $H$-clusters in quark matter, although giving wide ranges of parameters due to the uncertainty of $M_{\max}$. Figure 2 shows the dependence of $M_{\max}$ on $V_0$ and $\alpha_{BR}$, in the case $\rho_s = 2\rho_0$. To make comparison, we also plot the result when $\alpha_{BR} = 0$. The discrepancy between different values of non-zero $\alpha_{BR}$ is not very significant, and under a wide range of parameter-space $M_{\max}$ can be well above $2M_\odot$.

We derive the maximum mass of $H$-cluster stars to show that they could have safe maximum mass high enough to accord with the observations, although there are certainly some other kind of quark star models which provide possible ways to explain the observed high mass of the newly discovered pulsar PSR J1614-2230. However, it should be noted that the highest mass of pulsars that we find is surely different from the real maximum mass that a stable pulsar is able to have against gravity. The relation between measured masses of pulsars and their theoretical maximum mass could be compared to the case of white dwarfs, since the maximum mass of white dwarfs is well established to be about $1.4M_\odot$. The statistical study of nearby white dwarfs lying within 20 pc of the Sun shows that the distribution of measured masses of such sample of white dwarfs has a peak at around $0.6M_\odot$ and the most massive one is about $1.1M_\odot$ [36]. Assuming the same scaling for the case of pulsars, whose distribution of measured masses shows a peak at around $1.4M_\odot$ [37, 38], we could infer that the maximum mass of pulsars can be estimated to be $\sim 3.3M_\odot$ (using the peak value) or $\sim 2.5M_\odot$ (using $2M_\odot$ as the maximum value). The estimated maximum mass for pulsar-like stars would be $\sim 3.5M_\odot$, which is still much lower than the detected minimal mass ($\sim 5M_\odot$) of stellar black holes [39], if the mass ($2.74M_\odot$) of a pulsar (J1748-2021B) in a Galactic cluster is confirmed in the future. As shown by our results, $H$-cluster stars are consistent with the above estimation because their maximum mass could be $\sim 3M_\odot$ or even higher as long as the potential between clusters is deep ($-V_0 \gtrsim 100$ MeV). Therefore, discovering more massive pulsars in the future will certainly be helpful for us to get closer to the maximum mass and distinguish different models.

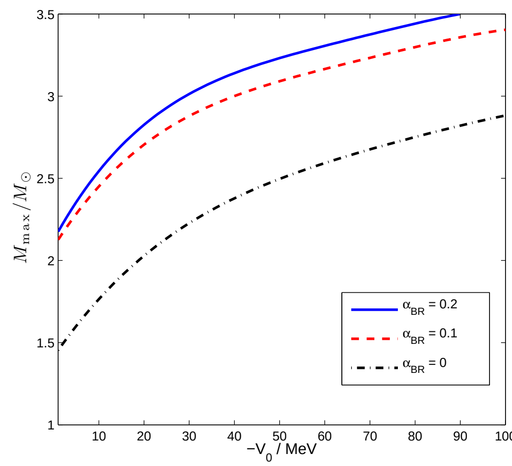


FIG. 2. (Color online) The dependence of $M_{\max}$ on $V_0$ and $\alpha_{BR}$, in the case $\rho_s = 2\rho_0$, including $\alpha_{BR} = 0.2$ (solid line), 0.1 (dashed line) and 0 (dot-dashed line).

## IV. DISCUSSIONS

### A. About the stiff equation of state

Composed of non-relativistic $H$-clusters with interaction in the form of Eq.(1), quark-cluster stars could have a stiff equation of state and a high maximum mass. Under some certain range of parameters, the equation of state could be so stiff that the adiabatic sound speed $c_s = \sqrt{dP/d\rho}$ is larger than the velocity of light. The probability that the speed of $c_s$ exceeding the speed of light in ultradense matter was studied extensively [40], and the issue regarding the causality and speed of sound was discussed in several theoretical points of view [41]. Microscopic theories consistent with special relativity prevent any real particle or signal moving faster than

light, but ultrabaric matter with speed of sound $c_s$ larger than speed of light is not necessarily superluminal [42].

In Newtonian hydrodynamics, the speed of $c_s$ is related to the thermal velocity and reflects the thermodynamic properties of the medium, so taking $c_s$ to be the signal propagation speed is meaningful. In the model that we use here, however, we do not consider the finite temperature effect, then the value of $c_s$ coming from Eq.(3) and Eq.(4) has nothing to do with the thermodynamic properties of the system and does not reflect the dynamics of the medium. Consequently, the adiabatic sound speed $c_s$ in our model is not a dynamically meaningful speed, but reflects the local stiffness. The interaction is mediated by mesons, so the real speed of interaction is obviously finite, and the signal propagation speed remains subluminal.

### B. The binding of $H$-clusters inside stars

At the highest density of the stars, which is about $6\rho_0$ in the case $V_0 = -10$ MeV and $3.5\rho_0$ in the case $V_0 = -100$ MeV (as shown in Fig. 1), the distance between two nearby $H$-clusters is about 0.8 fm. The size of $H$-dibaryons could be a little larger than that of nucleons, and then at the center of the star they could be so crowded that they touch the nearby ones, but they should be safe against being crushed. We assume that the touch of nearby $H$-clusters does not influence our overall picture, since it only happens at the very center of the star and the degree of touch is not high to cause dissociation.

In fact, the dependence of binding energy of quark-clusters on density is still unknown. However, if the mass of $H$-dibaryons decreases with increasing densities like baryons and mesons, this could be equivalent to the increasing of binding energy of $H$-dibaryons. At densities beyond $\rho_0$, the degrees of freedom become complex due to the non-perturbative nature of QCD, which could be responsible to the different binding behavior to the case at densities below $\rho_0$.

### C. Minimal mass of $H$-cluster stars

The formation of a $H$-cluster star in the collapsing process of a normal star could be energetically favored if the mass of the system is high enough. The minimal mass of $H$-cluster stars could be derived when the gravitational energy gained in going over to $H$-clusters can compensate the loss of rest-mass energy [43]. We consider the reaction $2n + 2\pi \to 2\Lambda \to H$, $H$-clusters could also form via the reaction $2n + 2\gamma \to 2\Lambda \to H$, but the energy of photons is difficult to quantify, so we omit such reaction, and the following is a rough estimation.

The energy defects in creating one baryon in $H$-cluster matter from nuclear matter via $2n + 2\pi \to 2\Lambda \to H$ is about 30 MeV, ignoring the $H$-$H$ interaction, so for a star with mass $M$ the total energy defects is $E_d \sim 0.5 \times 10^{53}\mathrm{erg}(M/M_\odot)$. Assume that the $H$-cluster star has a constant density $2\rho_0$, then the gravitational energy of the homogeneous sphere of mass $M$ is $E_g \sim -1.7 \times 10^{53}\mathrm{erg}(M/M_\odot)^{5/3}$ in Newtonian gravity. A star composed of $H$-clusters is more stable than that composed of nuclear matter when $E_d < -E_g$, that is $M > 0.16M_\odot$. At a much larger mass, the general relativity effect should be taken into account, in which the effective gravity is stronger and then the conclusion would become firmer. Therefore when the mass of a star is much larger than this critical mass, the formation of $H$-clusters with higher densities is energetically favored. If the $H$-$H$ interaction is $\sim -50$ MeV, then the minimal mass drops to $\sim 0.01M_\odot$. When $H$-$H$ interaction is deeper than $-60$ MeV, the mass of $H$-cluster stars could be arbitrary small.

It is surely possible that there could be normal matter surrounding a self-bound $H$-cluster star, but initially the surroundings would not remain because of energetic exploding [44–46].

### D. Clustering quark matter

Quark-clusters could emerge in cold dense matter because of the strong coupling between quarks. The quark-clustering phase has high density and the strong interaction is still dominant, so it is different from the usual hadron phase, and on the other hand, the quark-clustering phase is also different from the conventional quark matter phase which is composed of relativistic and weakly interacting quarks. The quark-clustering phase could be considered as an intermediate state between hadron phase and free-quark phase, with deconfined quarks grouped into quark-clusters, and that is another reason why we take quark-cluster stars as a special kind of quark stars. $H$-cluster stars are self-bound due to the interaction between clusters, with non-vanishing surface density but vanishing surface pressure.

It is worth noting that, whether the chiral symmetry broken and confinement phase transition happen simultaneously inside compact stars is a matter of debate (see [47] and references therein), but here we assume that the chiral symmetry is broken in quark-clustering phase.

### E. From the asymmetry term to a flavor symmetry

It is well know that there is an asymmetry term to account for the observed tendency to have equal numbers of protons ($Z$) and neutrons ($N$) in the liquid drop model of the nucleus. This nuclear symmetry energy (or the isospin one) represents a symmetry between proton and neutron in the nucleon degree of freedom, and is actually that of up and down quarks in the quark degree [48]. The possibility of electrons inside a nucleus is negligible because its radius is much smaller than the Compton wavelength $\lambda_c = h/m_e c = 0.24\mathring{A}$. The lepton degree

of freedom would then be not significant for nucleus, but what if the nuclear radius becomes larger and larger (even $\gg \lambda_c$)?

Electrons are inside a large or gigantic nucleus, which is the case of compact stars. Now there is a competition: isospin symmetry favors $Z = N$ while lepton chemical equilibrium tends to have $Z \ll N$. The nuclear symmetry energy $\sim 25(Z-N)^2/A$ MeV, where $A = Z + N$, could be around 100 MeV per baryon if $N \gg Z$. Interesting, the kinematic energy of an electron is $\sim 100$ MeV if the isospin symmetry keeps in nuclear matter. However, the situation becomes different if strangeness is included: no electrons exist if the matter is composed by equal numbers of light quarks of $u$, $d$, and $s$ with chemical equilibrium. In this case, the 3-flavor symmetry, an analogy of the symmetry of $u$ and $d$ in nucleus, may results in a ground state of matter for gigantic nuclei. Certainly the mass different between $u$, $d$ and $s$ quarks would also break the symmetry, but the interaction between quarks could lower the effect of mass differences and try to restore the symmetry. Although it is hard for us to calculate how strong the interaction between quarks is, the non-perturbative nature and the energy scale of the system make it reasonable to assume that the degree of the light flavor symmetry breaking is small, and there is a few electrons (with energy $\sim 10$ MeV). Heavy flavors of quarks ($c$, $t$ and $b$) could not be existed if cold matter is at only a few nuclear densities.

The above argument could be considered as an extension of the Bodmer-Witten's conjecture. Possibly it doesn't matter whether three flavors of quarks are free or bound. We may thus re-define *strange matter* as cold dense matter with light flavor symmetry of three flavors of $u$, $d$, and $s$ quarks.

## V. CONCLUSIONS

We propose in this paper that the strong interaction between quarks inside compact stars renders quarks grouped into a special kind of quark-clusters, $H$-clusters, leading the formation of $H$-cluster stars. Although there are many uncertainties about the stability of $H$-cluster matter, it could be possible that at high densities $H$-cluster matter is stable against transforming to nucleon matter.

The equation of state of $H$-cluster stars is derived by assuming the Yukawa form of $H$-$H$ interaction under meson-exchanges, and the in-medium effect from Brown-Rho scaling law of meson-masses is also taken into account. $H$-cluster stars could have stiff equation of state, and under a wide range of parameter-space, the maximum mass of $H$-cluster stars can be well above $2M_\odot$. Furthermore, if we know about the properties of pulsars from observations, we can get information on $H$-$H$ interaction; for example, if a pulsar with mass larger than $3M_\odot$ is discovered, then we can constrain $-V_0$ to be larger than 40 MeV. On the other hand, the mass of $H$-cluster stars could be as low as $\lesssim 0.1M_\odot$.

Although the state of cold quark matter at a few nuclear densities is still an unsolved problem in low energy QCD, various pulsar phenomena would give us some hints about the properties of elemental strong interaction [7], complementary to the terrestrial experiments. Pulsar-like compact stars provide high density and relatively low temperature conditions where quark matter with $H$-clusters could exist. $H$-cluster has been the subject of many theoretical and experimental studies. It is in controversy that whether $H$-cluster is a bound state, which depends on the quark-masses [49]. Whether quark matter composed of $H$-clusters could achieve at supranuclear density is still unknown, and on the other hand, the nature of pulsar-like stars also depends on the physics of dense matter. These problems are essentially related to the non-perturbative QCD, and we hope that future astrophysical observations would test the existence of $H$-cluster stars.

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